Reasons for relativistic mass and its influence on Duff's claims that dimensionful quantities are physically nonexistent

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Abstract. The main argument against the relativistic mass is that it does not tell us anything more than the total energy tells us, although it is not incorrect. But, one reason is to show a relation between time dilation and relativistic mass. This relation can be further used to present a connection between space-time and matter more clearly, and to show that space-time does not exist without matter. This means a simpler presentation than is shown with Einstein's general covariance. Therefore, this opposes that special relativity (SR) is only a theory of space-time geometry. The next reason is to show, how phenomenon of increasing of relativistic mass with speed can be used for a gradual transition from Newtonian mechanics to SR. The postulates, which are used for the definition of SR, are therefore still clearer and the total derivation of the Lorentz transformation is clearer. Such derivation also gives a more real example and counter-arguments for the debate regarding Duff's claims of physical nonexistence of dimensionful units and quantities (PND). Therefore, the debate about Duff's claims becomes clearer. Still other counter-arguments against PND are added.

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1. Introduction

An understanding of the elementary physical theories and of their various aspects is important both for students and for researchers of still undiscovered theories, such as quantum gravity. Relativistic mass ($m_{\rm r}$) is used here for a different interpretation of the theory of special relativity (SR). The shortest way to define $m_{\rm r}$ is:

$$m_{\rm r} = W/c^2 \,, \tag{1}$$

where W is the total energy and c is the speed of light. One argument against using $m_{\rm r}$ is that it confuses students. The main argument against using $m_{\rm r}$ is that there is no sense to do so [2, 3, 4, 5], as it does not tell us anything more than W tells us; although it is not incorrect. But, in the next article the reasons will be shown that this is not true:

- One reason is to show a relation between time dilation and $m_{\rm r}$.
- This relation can be further used to present a connection between space-time and matter more clearly, and to show that space-time does not exist without matter. This means a simpler presentation than it is shown with Einstein's *general covariance* [6, page 847].
- The next sense is to show, how the increasing of $m_{\rm r}$ with speed can be used for a gradual transition from Newtonian mechanics to SR.
- The modified postulates of SR additionally clarify SR and the derivation of the Lorentz transformation.
- Such derivation gives a more realistic example for debate on Duff's claims [7, 8] for physical nonexistence of the physical units are the dimensionful constants (PND). Thus, the debate about Duff's claims becomes clearer.
- Other reasons can be found in [3, 9] and in references therein.

In section 2 it si shown somewhat a different derivation of SR, where $m_{\rm r}$ is used. It is evident also, how insight in SR is clearer. In section 3 it is shown how space-time and matter are connected. It is shown with the common interpretation of SR and with this interpretation that space-time does not exist without matter. This connection is also shown from other aspects. In section 4 it is shown influence of $m_{\rm r}$ on Duff's claims that for PND. Duff's claims are widely acknowledged in science community, but everything is not clear. One of Duff's provocative questions is also, why are there precisely three elementary units (kg, m and s), why not less or more. Some clarifications or contra-arguments will be given in this paper. Only a part of its claims is discussed here.

2. Derivation with use of $m_{\rm r}$

Let us imagine a trolley inside of a moving rocket that moves perpendicularly to the direction of the rocket. When the rocket increases velocity, the trolley moves a little bit slower than before (according to a standstill observer). The common explanation is that a cause is time dilation. But, an explanation also can be that the transversal momentum is constant with velocity, hence increasing of m_{r} means smaller velocity of such a trolley. Such explanation shows a relation between m_{r} and time dilatation. Such a relation also confirms that space-time does not exist without matter.

Essential difference of such interpretation of SR with the common interpretation of SR is that the following transformations are used:

$$m = m_{\mathrm{r}}/\gamma\,, \tag{2}$$

$$t' = t''/\gamma\,, \tag{3}$$

where m is mass of the trolley inside of the rocket, and t' is transformed time inside of the rocket which is obtained with the common Lorentz transformation,‡ and γ is defined as

$$\gamma = \left(1 - \left(\frac{v}{c}\right)^2\right)^{-1/2} \tag{4}$$

with v the velocity of the rocket. Of course, it is understandable that physics is the same if these transformations are used. This transformation already gives that the trolley inside of the rocket moves slower because of conservation of the momentum in the transversal direction.

But, let us show a detailed derivation in order how things will be clearer. We will also see how input postulates can be simplified. Einstein's postulates of the SR are

(i) The laws of physics are the same for all observers in an inertial state of motion.

(ii) All inertial observers always measure the speed of light as being the same.

Let us add to these two postulates still two known, acceptable postulates

(iii) Space is isotropic for all observers.

(iv) A maximal speed inside of every inertial system is speed of light c'' (which is not necessarily equal to c).

But, let us omit postulate (ii). The reason for the ommision of this postulate is that (ii) will be derived in the following paragraphs. Otherwise it seems to the autor that this is a slightly more empirical postulate than the other three ones, and it is less self-evident.

Let us synchronize clocks in another inertial system so that we see them to move with the same rate as our clocks. Of course, this does not mean synchronisation in the opposite direction. Therefore, speed of light in the transversal direction regarding an observer in a rocket (with respect of the our transformations) can be simply calculated. An observer from the stationary system sees the speed of light equal to c, of course. He can use Pythagoras' theorem, and therefore he calculates that the observer in the rocket sees transversal speed of light c''_{trans} equal to

$$c''_{\mathrm{trans}} = c/\gamma\,. \tag{5}$$

‡ Time for a standstill observer is commonly assigned as t.

Postulate (iii) also gives that the longitudinal speed of light equals to the transversal speed of light, this means

$$c''_{\text{long}} = c/\gamma\,. \tag{6}$$

Because of equal status of both inertial systems, an observer in a moving system must calculate γ in (4) with the same value, but this gives that he sees smaller velocity of the inertial system (v'') than it is seen by the first observer:

$$v'' = v/\gamma\,, \tag{7}$$

therefore, from his point of view all velocities are proportionally smaller.§ It is said for another observes that he sees the same γ, but not the same c. This is one step closer to calculation with dimensionless numbers [7, 8], which are physically more fundamental.

The derivation of the *Lorentz transformation* can be started in the similar way as in the common derivation of SR; this means that we begin with the two initial equations of the Lorentz transformation:

$$x'' = \eta(x - vt)\,, \tag{8}$$

$$x = \eta(x'' + v''t'')\,, \tag{9}$$

where x and t are space and time coordinates of a stationary system and x'' and t'' are space and time coordinates of a moving system (for instance, a rocket), which moves with velocity v in the direction x.‖ η means a factor of Lorentz contraction and it still needs to be calculated by applying (6) and (7).¶ Hence a calculation gives that η equals γ. These equations also give how time is transformed:

$$t'' = \gamma^2(t - vx/c^2)\ , \tag{10}$$

$$t = t'' + v''x''/c''^2\ . \tag{11}$$

Now let us respect that everything in the moving system is moving slower, hence also processes in brains and also its clocks. Therefore, we can use the transformation (3). (3) together with (6) to (11) gives back the common equations of the Lorentz transformation and hence gives equal speed of light in all inertial systems. So, this is a transition from the above three postulates (i), (iii) and (iv) to the common two postulates (i) and (ii). But, further analyses will be done with t'' as it is calculated in (3).

The smaller speed of light in (5) and (6) (and proportionally smaller all velocities, for instance (7)), can be compensated by larger mass, so the momentum in the transversal direction is preserved.

(10) and (11) are less symmetric, because factors before (8) to (11) are γ, γ, γ^2, and 1, but in the common derivation they are all equal to γ. But, we can see some simplifications:

§ If this was not true, the velocity of the first system v'' would exceed c'', what is against postulate (iv).

‖ If we are more precise, then the minus sign can stand before $v''t''$ in (9) because, in truth, v'' means opposite velocity. In this way those four equations (8) to (11) become still more symmetric.

¶ The procedure with η (γ) is also used in the common calculation of the Lorentz transformation.

- simplifications of the postulates (i) and (ii),
- simpler and clearer calculation of γ,
- dimensionless numbers are more frequently used,
- presentation with momentum is clearer, (what will also be seen further),
- we can use the minus sign for velocity v''.

Now it is seen, how it is with conservation of the momentum in the transversal direction. Let us see still more clearly, how it is with movement in the longitudinal direction. Let us look at, how it is with increasing of W with velocity. This can also be additionally clarified with use of $m_{\rm r}$. Therefore this is a further visualisation of these equations.

For the beginning, let us naively assume that space is Euclidean, and that acceleration increases W and hence also $m_{\rm r}$. Then the equation for increasing of energy of an accelerating body is:

$$\mathrm{d}W = c^2\mathrm{d}m_{\rm r} = m_{\rm r}a\mathrm{d}x = m_{\rm r}v\mathrm{d}v\,, \tag{12}$$

where a means acceleration, x means distance and v means velocity. A solution of (12) is

$$2\ln(m_{\rm r}/m_{\rm r0}) = (v/c)^2\,, \tag{13}$$

where $m_{\rm r0}$ was mass at velocity zero, and ln is the logarithm with base e. The result is wrong, because the real relation is

$$m_{\rm r} = \gamma m_{\rm r0}\,. \tag{14}$$

But, additional supposition should be that longitudinal distances x_1 in the rocket seen from the rocket are larger than the same distances x seen from the rest system. (This is *Lorentz contraction*.) This can also be comprehended from (8) to (11):

$$\mathrm{d}x_1 = \gamma\mathrm{d}x\,. \tag{15}$$

If this is corrected in (12), the new equation is:

$$c^2\mathrm{d}m_{\rm r} = m_{\rm r}\gamma a(\gamma\mathrm{d}x) = m_{\rm r}(\gamma v)\gamma\mathrm{d}v\,. \tag{16}$$

and the result is (14), what is correct. The equation, similar to (16) is known from the common calculations of SR:

$$\mathrm{d}W/\mathrm{d}t = \gamma^3 mva\,. \tag{17}$$

It is interpreted that longitudinal relativistic mass ($m_{\rm rt}$) equals

$$m_{\rm rt} = \gamma^3 m_{\rm r0}\,. \tag{18}$$

But, in the present example the part γ^2 is attributed to length-contraction and not to $m_{\rm rt}$.

Therefore it is obtained with the use of acceleration, how W expressed with $m_{\rm r}$ increases with increasing of velocity.

- Thus, properties of the mass ($m_{\rm r}$), such as inertia or rest, are also important, not only properties of its energy counterpart (W).
- At the same time, this is also a gradual transformation from Newtonian mechanics to SR.

The interpretation with $m_{\rm r}$ is not the only one which is different as the common interpretation of SR. It can be respected that G, $\hbar$ and c are changed so as time in neighbouring inertial system is changed, or that G, c and m_is are changed in neighbouring inertial system, for instance.

3. A connection between matter and space-time

A remark is possible that we see larger mass inside the rocket, but from the rocket they see us that we have larger mass. Therefore it seems that increasing of mass is not realistic. But, this is exactly the same problem as in the common Lorentz equations, where relations for time show the same paradox. But, SR is a correct theory, and both the common interpretation of SR and the interpretation with $m_{\rm r}$ are correct. In short, it is not incorrect that relation mass-time is a one-way one. It is important that the introduction of $m_{\rm r}$ explains dilatation of time and connection between matter and time. It is not necessarily to look at the same time from two inertial systems, but it is enough to look at once from one inertial system and another time from another inertial system.

This slower speed of time with increasing of $m_{\rm r}$ can also be generalized out of SR to big and small elementary particles. If a human body was made from the same particles, but 1000 times lighter ones, the speed of time would seem to us much smaller than now. Hence one second would seem very long. This example is not relative, because it gives the same result from both observers.

This can be generalized still further. A fly feels a longer second than an elephant, because of smaller mass of the fly brain the brain processes are faster. Although particles are not smaller, this also can be an analogy for reduction of the "mass". Another example from biology is a cold lizard or a warm one. For the first one a second seems shorter. Therefore various examples of different time flows have a very similar key foundation, this is the momentum or movement.

We know from the common interpretation of SR that rest matter cannot be accelerated to $v = c$. It can only be approached to this speed. But, anywhere close to c this matter is moving, always we can find an inertial system, where this matter is at rest. The speed of a photon equals c. We cannot find an inertial system where it is at rest. Time flows where rest matter exists, but time does not flow for a photon. Therefore rest matter defines that time flows; hence that time is dependent of rest matter.

Interpretation of SR with $m_{\rm r}$ tells us still more. It tells us that speed of time is dependent from largeness of mass. Therefore this is another clarification that space-time does not exist without rest matter. Hence, this is a simplified explanation of what it is shown with general covariance [6, page 847].

Hence formally, one time is really attributed to every point of space, but truly time flows only if rest matter is present, or differently said, that matter is a reference for this space-time. Therefore space-time without rest matter does not exist.

But one detail should be clarified still. Seemingly, time flows for a photon

- because it has some frequency,
- if it is calculated for rest matter that time does not flow at $v = c$, this does not mean automatically that time does not flow for a photon. Precisely said, it means only that time does not flow for rest matter if it is accelerated to $v = c$. And, of course, it never reaches this speed.

Indeed, frequency of a photon is dependent from rest matter, or from inertial system, where this matter is at rest. But, privileged inertial system does not exist. Therefore rest matter cannot be ignored where the existence of photons is mentioned. Thus, photons exist because of rest matter. It is similarly in general relativity, where it is claimed that gravitational waves exist independently of matter. But indirectly they are connected with matter.

Connection between elementary particles and space-time was indirectly found also by Cramer's Transactional interpretation of quantum mechanics [10]. Namely, a photon does not fly into empty space, but checks with "hand-shaking" if there is any other particle, and then flies.

Let us think in the approximation, where space-time is continuous, therefore it is not grained. In the continuous area, points can never build up a straight line, because always is valid $0 \times x = 0$, despite even when $x = \infty$.[+] Therefore three-dimensional space can be partitioned to smaller pieces of three-dimensional space, but it cannot be partitioned to two-dimensional objects. Hence space-time is not built up from rest pictures of space, but it also includes time transition between these pictures. Therefore we always have "five dimensional volume", hence matter is another "dimension". But it seems that space-time is not continuous, but is grained. Despite of this, the smallest cell also implicitly includes matter.

Oas [5] commented that acceleration gives rise to energy but not to a larger number of particles. (So, by him, m_r does not tell us anything new according to energy.) But, this also tells us that m_r is increased inside of elementary particles, therefore elementary particles are these essential things.[*] So, dimensionless numbers μ_i are generators of all space-time. $\mu_i^2 = m_i^2 G/(\hbar c)$, where m_i are masses of various elementary particles, G is the Newton gravitational constant, and $\hbar$ is the Planck constant. Fotini Markopoulou claims similarly [11]. Hence again, elementary particles are necessity for the existence of space-time.

Hence energy shows a property of matter, this is inertia, what is expressed with the momentum. Energy also shows another property of matter that can be at rest. Of

[+] This is different as limits, for instance, $\lim_{x\to 0} \sin(x)/x = 1$, because $x = 0$ is never reached.

[*] Rest matter is built up from elementary particles, but it can be built up also from black holes. Maybe those two things are the same. The answer is hidden in a quantum gravity theory.

course, energy inside of photons is not at rest, but regarding all above, space-time does not exist without rest matter, therefore energy of photons also does not exist without rest matter.

4. Influence of $m_{\rm r}$ on Duff's claims

Duff [7, 8] claims for PND, but Okun [7] claims contrary. One of provocative Duff's questions is also why precisely three elementary units (m, s and kg) exist, why not, for instance, two or seven ones. But, in the example above it is evident that those three units form almost a complete set, because time does not exist without matter and it is part of space. We can also see that this is connected with conservation of the momentum. At the same time we can conclude, that the "Cube theory" [7] is not an appropriate answer for Duff question and that its connection with Duff's question is only an accident.

The common interpretation of SR assumes that c is equal inside of all inertial systems. This agrees with Okun's supposition [7] that dimensionful quantities are also physically important, not only dimensionless quantities. But, the above derivation shows an example, where constancy of c is no more so important.

We can also include the mentioned example where masses of all particles are 1000 times smaller. In this case c would seem much smaller to us. Therefore constant c is valid only if μ_is are constant and also if the common interpretation of SR is used. Hence these two conditions considerably lower universality of constancy of c. Of course, constancy of c is a consequence of relativity of inertial systems. So, constancy of c is still ever physically important. The both interpretations also show that foundation of everything is rest matter. Hence also such an efficiency of the common interpretation of SR with the constant c. The connection of matter and space-time in this paper and description of μ_is as carriers of everything help us to debate about the importance of dimensionful quantities.

So, the example of derivation with $m_{\rm r}$ is more realistic, because it really exists, it has some purpose and it is more imaginable than Duff's examples [7, 8]. It is also shown that dimensionful quantities (as c) are also relative. In the common interpretation of SR, c is such as it is felt by an observer in any inertial system. The interpretation of SR with $m_{\rm r}$ gives c in a neighbouring inertial system, as it is felt by observer in our inertial system. At examples with the small particles and with the animals, the speed of light is such as it is felt by an observer in this inertial system. But, speed of light c''' calculated by Schrödinger's units

$$c''' = c/\alpha \tag{19}$$

is not felt by anyone. This example in unnatural, forced and with small realistic purpose.

Duff's main objection is that dimensionless quantities occur in physical calculations, and that dimensioful quantities are only mathematical tools, but they are not physically real. This can be answered with a comparison with the quantum mechanics. Let us look at, for instance, Feynman's derivation in [12]. Complex amplitudes here are summed up, and probability for one event is proportional to the square of this sum. Differently said, their linear impact is important. We can ask ourselves, if those imaginary numbers are physically real. In the above example they are indirectly physically real, because they are used in mathematical procedure. Use of imaginary numbers means also that the partial result of calculation is not physically real. If any derivation without imaginary numbers exists, still ever it will be a conclusion that a partial result is not physically real. One example of a view from a different aspect is Zeilinger-Brunkner's one [13].

The physical units behave similarly as amplitudes. For instance:

$$ln(2kg) - ln(1kg) = ln(2) - ln(1) + ln(kg) - ln(kg) = ln(2) - ln(1)\,. \tag{20}$$

$ln(kg)$ is not defined (is not directly physically real), but it cancels in the calculation, similarly as imaginary numbers cancel with squaring. Duff has also not shown any example where the physical units are not necessary to use in a derivation.

Duff also asserts for PND; this means also physical existence of c, G and $\hbar$. c means one relation between time and length. Therefore analogically, we can say that the radian is approximately a relation between rectangular lengths, the analogous question is for PND of the radian. But the answer is that it exits physically.

Of course, the above examples do not finish the debate in the trialogue. One of key Duff's arguments is that dimensionful constants are redundant regarding dimensionless constants μ_is and α. An answer on this one demands a special article. But, in short, physics needs more parameters than only dimensionless particle masses (μ_is) and charge (α). He also used examples with variable dimensionless and dimensionful elementary constansts [7, 8], what need more precise treatment. This article can also be introduction for more precise treatment of Duff's claims.

5. Conclusion

It is not easy to visualize SR with only $c = constant$. But the additional presentation with m_r helps us to visualize it better. The presentation with m_r together with a more precise common interpretation of SR also opposes that SR is only the theory of space-time geometry. It is not enough to show the symmetry of four-momentum to space-time, it should also be shown why this symmetry exists. The connection of space-time and mass shows that kg, m and s form one triplet. This is used to oppose Duff's claims for PND.

For a *theory of everything* we need to go to foundations. The postulates are also foundations of physics, not only formulae. A property of the equations in SR is that they are hyperbolic. But this is a consequence of the postulates, it is not a fundamental property. Some dimensionless constants are also foundations of physics. It is beneficial that they appear already in pre-theories of theory of everything, such as in SR.

Such many sided interpretations of formulae should also be written in other fundamental physical theories, for instance in quantum mechanics. One example, where this is done, is Zeilinger-Brukner interpretation of quantum mechanics [13]. Generalization of a new aspect of a known theory can give new cognitions.

The author is unsatisfied, because SR was not presented to him in school also in this way. This feeling is one example which shows on incompleteness of the common interpretation of SR. It is a trend in teaching of fundamental physics, that it should be as abstract as possible. For instance, it is known that our closed universe exists without any space out of universe. But, although external space does not exist, it is easier to visualize it. Similarly, it is better to visualize m_{r}.

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